Switch on STEM

MATHS

The Bean Machine

The bean machine is designed to demonstrate the normal approximation to the Binomial Distribution. Consider a series of events, or trials, each of which has two possible outcomes, success or failure, and each event is independent of the previous event (e.g. tossing a coin). The probabilities of getting a certain number of successes in the trials (from zero successes to each trial being successful) are described by the Binomial Distribution. As the number of trials increases, this distribution can be approximated by the Normal Distribution or ‘Bell Curve’.

Students are asked to flip a coin a set number of times and record the number of heads that they obtain. A series of column positions are created for the possible number of heads achievable i.e. zero to the total number of coin flips. After a student has counted the number of heads that they have gotten in the coin tosses, they will add a Lego block onto the appropriate column. After repeating this for a number of students, the distribution of the Lego blocks should resemble a Bell Curve.

Equipment Overview

This activity simulates the action of a ‘bean machine’, ‘Galton Board’ or ‘Quincunx’. It is possible to have one of these made, but it is expensive. A small plastic one is available, , but shipping and taxes increase the cost. Students will be shown a simulation or video of a bean machine working instead.

To simulate the machine each student takes one ‘bean’ and figures out where it ends up. The ‘bean’ is then placed in the appropriate column. The columns can be lines on a whiteboard, marked out by tape on a metal sheet or spaces on a Lego baseplate. The beans could be dots that are coloured in, magnets that are added or Lego blocks that are built up.

Equipment

- Model to create the Distribution:
  - Lego Duplo bricks +bases.
  - Or use a whiteboard – draw dots on for each trial.
  - Or a magnetic board/metal sheet with small magnets.
- Tokens for Random Walk
- Coins for flipping
- Worksheets

Equipment - Stand

- Tables – 1 for coin flipping and 1 for the model.
- Poster + Poster board/wall
- Laptop
- Power socket.
- Pens

Equipment - Costs

ITEM	PRICE	QUANTITY
LEGO DUPLO Basic Bricks Deluxe	25.00	4
LEGO DUPLO Baseplate 15”	13.00	2
Plastic Coins	9.00	1

Available directly from LEGO:
http://shop.lego.com/en-IE/LEGO-DUPLO-Basic-Bricks-Deluxe-6176?fromListing=listing
http://shop.lego.com/en-IE/Green-LEGO-DUPLO-Baseplate-2304?fromListing=listing

Prices not provided for paper, card, ink etc. Each institution will have its own Office Supplies provider or access to these supplies.

Plastic coins are available from Evans Educational:
http://www.evanseducational.ie/Euro-Coins-EED1-713/

Any items like buttons, plastic chips, sweets etc can be used as tokens.

Preparation

- Print Worksheets
- Download the bean machine simulation/video.
- Practice tossing coins.
- Familiarise yourself with the background information

Demonstration

This activity examines each of the following:

- 50-50 events
- Random walks
- Probability distributions
- Binomial Distribution
- Normal Distribution

Set Up

- Mount Poster
- Set up Laptop to show simulation of Bean Machine.
- Affix Lego Baseplates to the table
- Label the ‘columns’ for bricks to go on.

Explain Bean Machine

- Show students the bean machine (or picture) and explain how it works:
  - Each ball or ‘bean’ hits the first peg and bounces left or right.
  - It then hits a peg on the next row and bounces left or right again.
  - In each row the bean hits a peg and bounces left or right.
  - At the end the bean drops into one of the columns or ‘bins’.

Prediction: Question Students

- Where do you think the bean will land?
- Could the bean land in any column?
- Where is the most likely place for it to end?
- What do you expect will happen when all the beans drop?
- Will they all be in the same place? Or evenly spread out?
- Will they be concentrated at one end or the other?
- Sketch the shape on your handout!

Answers

- Any given bean can land in any column.
- But it is most likely to end in the middle – why?
  - Each path has the same probability.
  - Some columns have multiple paths leading to them
  - There are more ways/paths to the middle than to the edges.
- Therefore most of the beans will end up in the middle.
- But a few beans will still reach the edges!

The Experiment

"Today we are running an experiment! We are going to make a bean machine from Lego. You will each simulate the path of a bean. At the end of the day we will see what the shape looks like."

Give each student a coin, a token and a Lego brick.

Ask the students to follow the steps on the next slide

The Steps

- Toss the coin and note down if it was heads or tails.
  - If it was heads it counts as a ‘right’ for the bean so move your ‘bean’ one place to the right on the number line (+1).
  - If it was tails it counts as a ‘left’ so move your ‘bean’ one place to the left on the number line (-1).
- Repeat this until you have 20 tosses
  - Any even number can be substituted here for a given event.
- Place your brick in the corresponding column to your end position.

Background Information

- The Galton Board was invented by Sir Francis Galton to demonstrate that the normal distribution is approximate to the binomial distribution.
- It consists of rows of pins aligned in a pyramid shape, with a series of columns, or bins, beneath. Balls are allowed to fall through the rows of pins and settle in the columns below. While it is impossible to predict where an individual ball will land, we can however work out the probability that a ball will land in a particular bin.
- When a ball hits the first pin, there is a 50-50 chance that the ball will bounce the right or to the left. Say it goes right, it will then hit the pin below and to the right of the previous one. Once it hits this pin, there is again a 1 in 2 chance that the ball will bounce the right or to the left. This process continues until the ball bounces off a pin in the lowest row and falls into one of the bins.

Binomial Distribution

- Let's label the leftmost bin the 0^{th} bin, the next the 1^{st} bin and so on. If a ball bounces right *k* times (and left the other times) then it will finish up in the *k*th bin.
- The number of ways that a ball can bounce right k times in n rows of pins is given by 'n choose k': C^n_k, which is known as a **binomial** coefficient. In other words, there are C^n_k ways for the ball to end up in the *k*th bin. Given that the probability of bouncing right at any pin is 1/2, the probability that a ball will land into the *k*th bin after bouncing through *n* rows of pins is then given by

$$C^n_k \ (1/2)^n$$

Normal Distribution

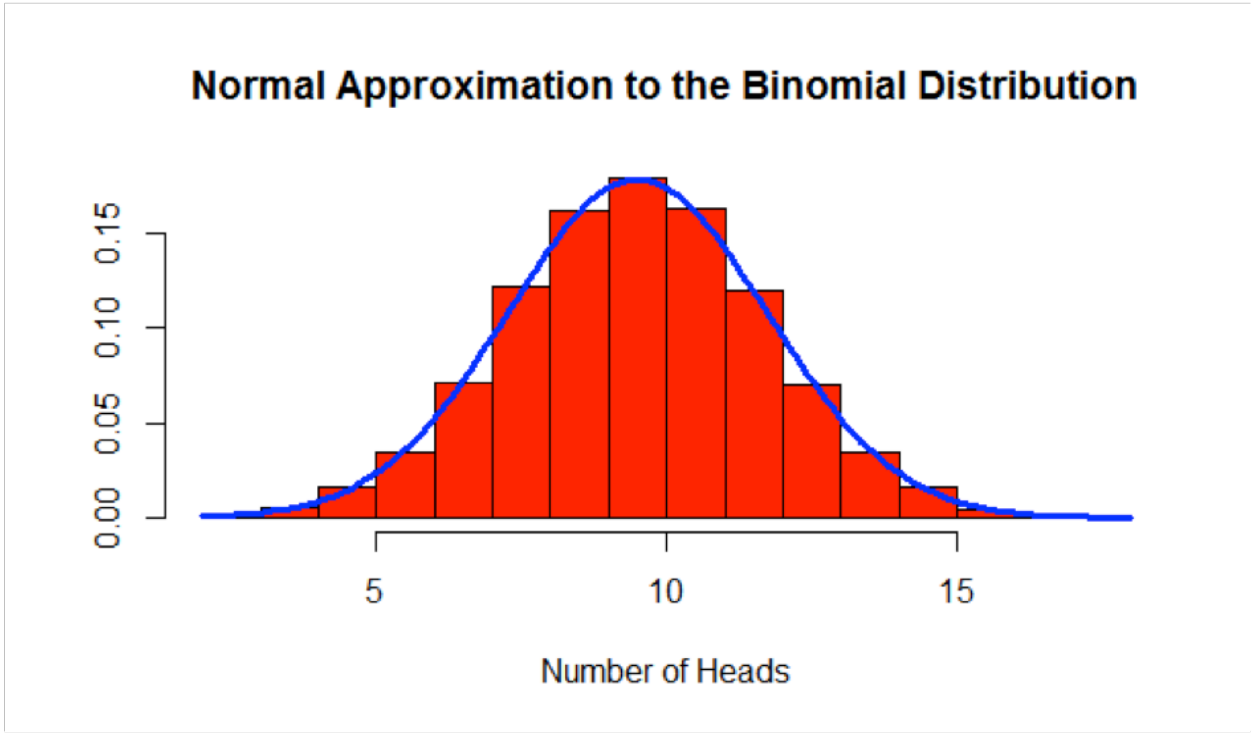
The Normal Distribution describes a continuous set of data (such as time, distance etc.) which is distributed symmetrically about the mean, or average, of the data. It is often called a 'Bell Curve' as it resembles the shape of a Bell.

The Normal Distribution is extremely useful as it closely approximates many distributions of data in everyday life. Eg.

- People's Heights
- American S.A.T scores
- People's Blood Pressure

These can all be approximated by the normal distribution. Also, as it is a continuous distribution, it is easier to work with mathematically than a discrete one such as the Binomial Distribution. It is for these reasons that we often make use of the **Normal Approximation**

If we once more consider a Binomial Distribution with n trials, the normal approximation says that, under certain conditions, we may approximate this using the Normal Distribution. Roughly speaking, the larger n (the number of trials) is the better approximation we obtain



Links

Galton board, visual only:

https://www.youtube.com/watch?v=p65aYYuAz-s

Explanation by the IMA:

https://www.youtube.com/watch?v=6YDHBFVIvIs -

More comprehensive video covering binomial v normal

https://www.youtube.com/watch?v=CCqWkJ_pqNU